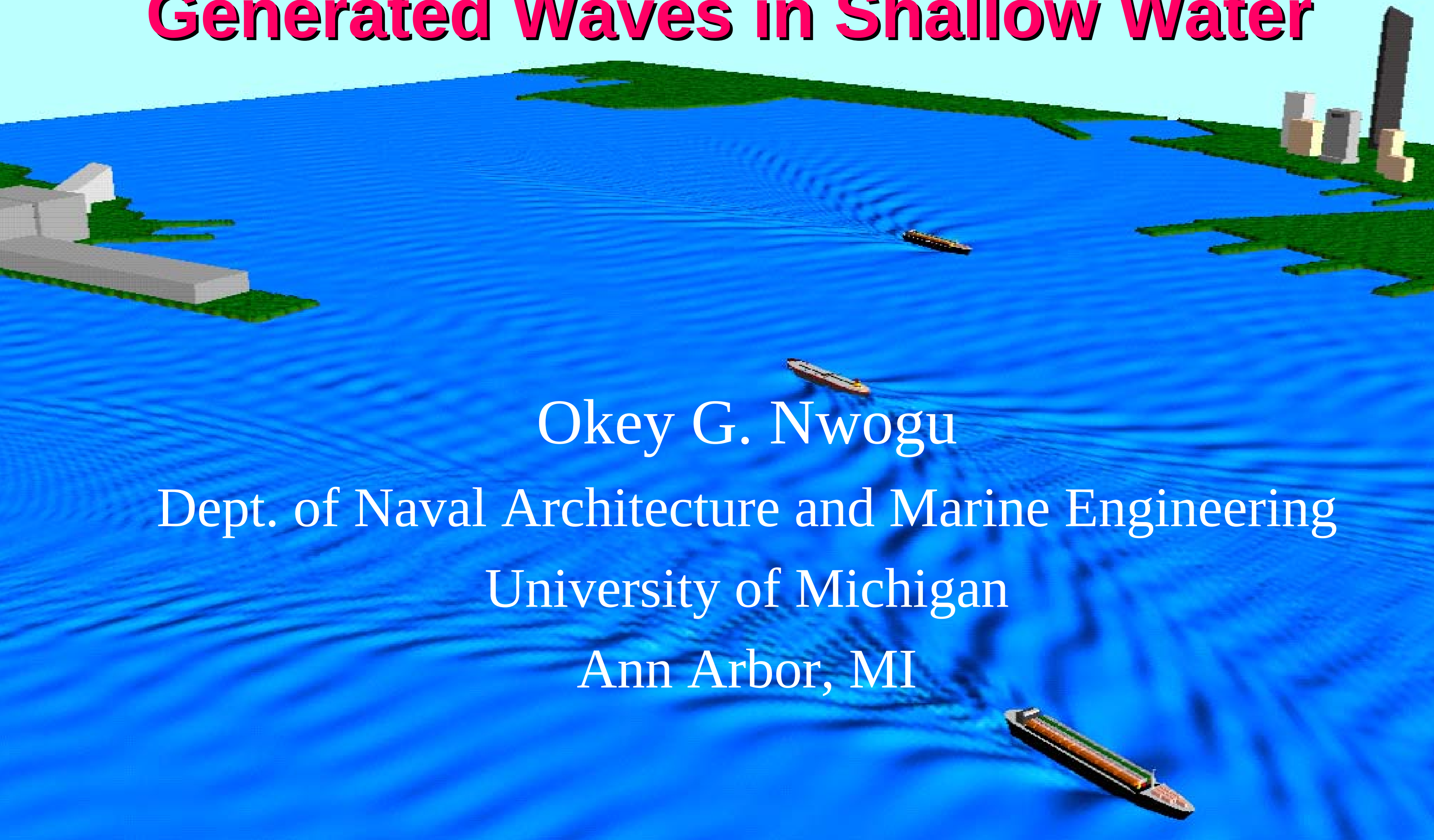
Boussinesq Modeling of Ship-Generated Waves in Shallow Water



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New Scientist Magazine - Aug. 1999

- *Solitary Killers - "The sea was calm when two friends took their boat out for a quiet spot of fishing. In the distance, a high-speed catamaran passed them. Ten minutes later, a huge wave 5m high came out of nowhere and swamped their 11m boat. Sibley was thrown overboard and drowned. Hayman later told reporters that the wave looked like the white cliffs of Dover"*

Complaints - Ship-Generated Waves

- *Bank stability and shoreline erosion*
- *Potential impact on marine life in coastal wetlands*
- *Risk to smaller boats using waterways*
- *Swimmer safety*
- *Berthing/mooring and On-Offloading difficulties for cargo vessels in inland ports*
- *Hazard to recreational boat marinas*

New-Generation Vessels

- *High-Speed Vessels*
  - *Speeds of 35 to 50 knots compared to 10 to 15 knots for conventional vessels*
- *Deep Draft Vessels*
  - *Post-Panamax Class of Vessels*
  - *Length=O(1000ft), Beam=O(150ft), Draft = O(50ft)*

Modeling Approaches

- ***Far-Field***
  - ***Shallow Water Equations***
  - ***Boussinesq Equations***
- ***Near-Field***
  - ***Boundary-Integral Methods***
  - ***Navier-Stokes Equations***

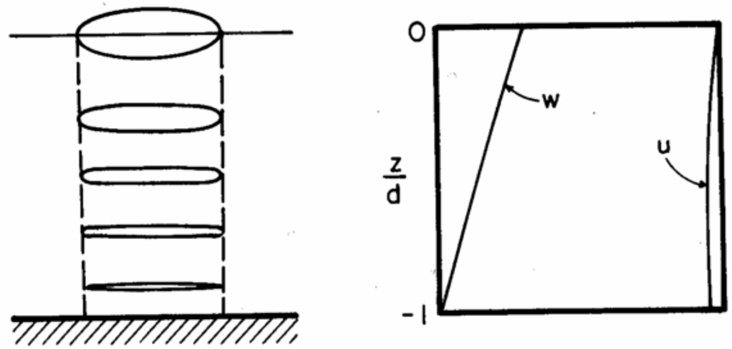
Far-Field Methods

- ***Shallow Water Equations***
  - *hydrostatic pressure*
  - *nondispersive waves*
  - *better suited for estimating water level drawdown/return currents*
- ***Boussinesq Equations***
  - *non-hydrostatic pressure*
  - *able to deal with shorter-period dispersive waves*
  - *poor description of flow field around ship hull compared to near-field methods*

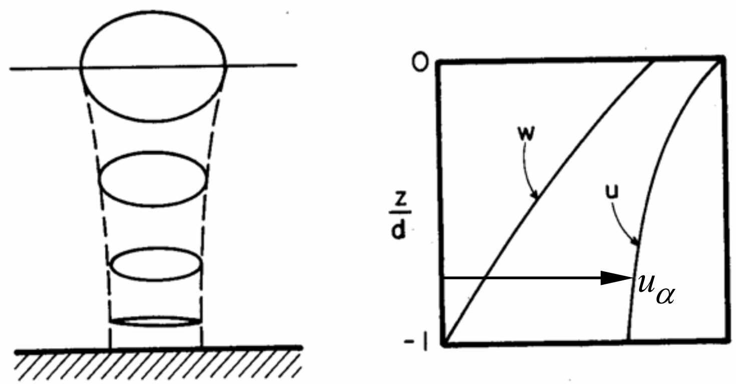
Near-Field Methods

- ***Boundary-Integral Methods***
  - *assumes irrotational flow*
  - *accurate description of flow around ship*
  - *computationally intensive for large areas when you have to discretize seabed in addition to body and free surface*
- ***Navier-Stokes Equations***
  - *able to deal with viscous effects*
  - *accurate description of flow around ship*
  - *computationally intensive for large areas*

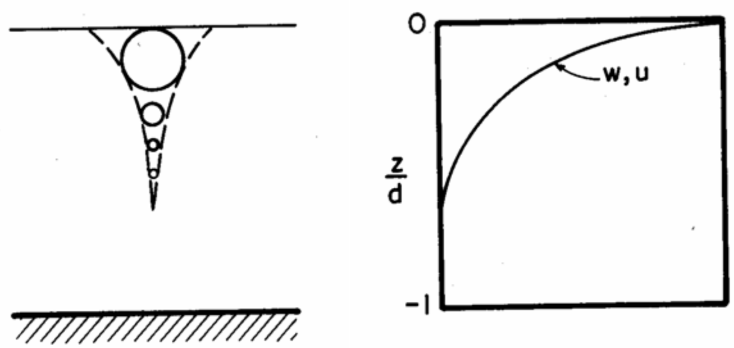
Wave Orbital Variation with Depth



(a) Shallow water



(b) Intermediate depth



(c) Deep water

Boussinesq Equations

- *Taylor series expansion of velocity potential around arbitrary elevation z_α in water column*

$$\Phi(x,y,z,t) = \phi_\alpha(x,y,t) + (z_\alpha - z)\nabla \bullet (h\nabla\phi_\alpha) + \frac{1}{2}(z_\alpha^2 - z^2)\nabla^2\phi_\alpha$$

  - *quadratic variation for horizontal velocities and hydrodynamic pressure*
  - *linear variation for vertical velocity*
- *substitute into 3-D mass and momentum equations and integrate over depth*

Boussinesq Equations

- *Mass Conservation Equation*

$$\eta_t + \nabla \cdot \boldsymbol{Q} = 0, \quad \textit{where} \quad \boldsymbol{Q} = \int_{-h}^{\eta} \boldsymbol{u}\, dz$$

- *Momentum Equation*

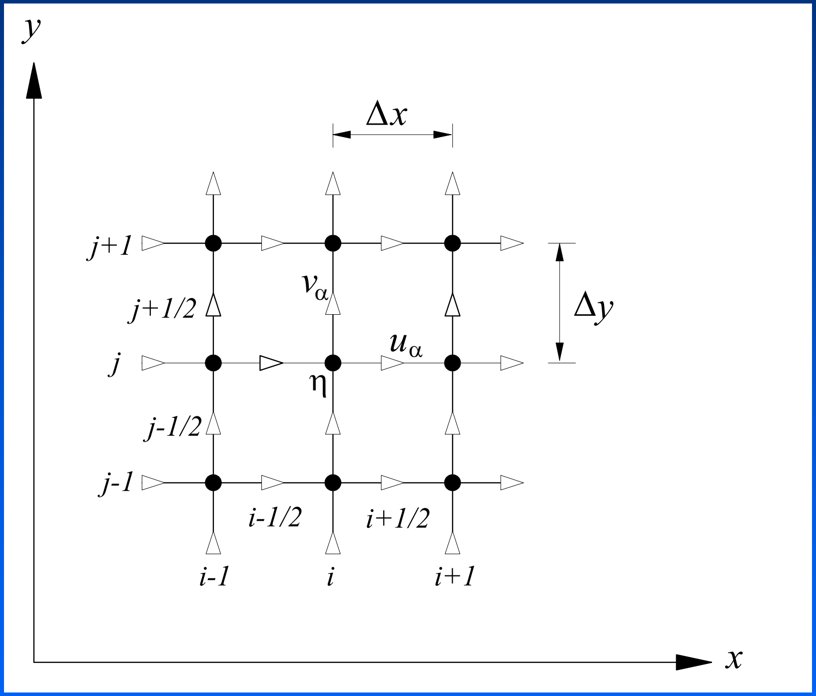
$$\boldsymbol{u}_{\alpha,t} + g\nabla\eta + (\boldsymbol{u}_\alpha \cdot \nabla)\boldsymbol{u}_\alpha + \left[\frac{z_\alpha^2}{2} \nabla(\nabla \cdot \boldsymbol{u}_{\alpha,t}) + z_\alpha \nabla\left[\nabla \cdot (h\boldsymbol{u}_{\alpha,t})\right] \right] = 0$$

- *Ship represented by line source (Tuck, 1966)*

$$\bar{v}\Big|_{y=0} = \pm \frac{U_s}{2h} A_x$$

Numerical Solution

- *Time-domain, finite difference method (modified Crank-Nicolson)*
- *Staggered Grid*
- *Line by line technique for momentum equations resulting in tridiagonal matrices for u_α, v_α*



Practical Limitations

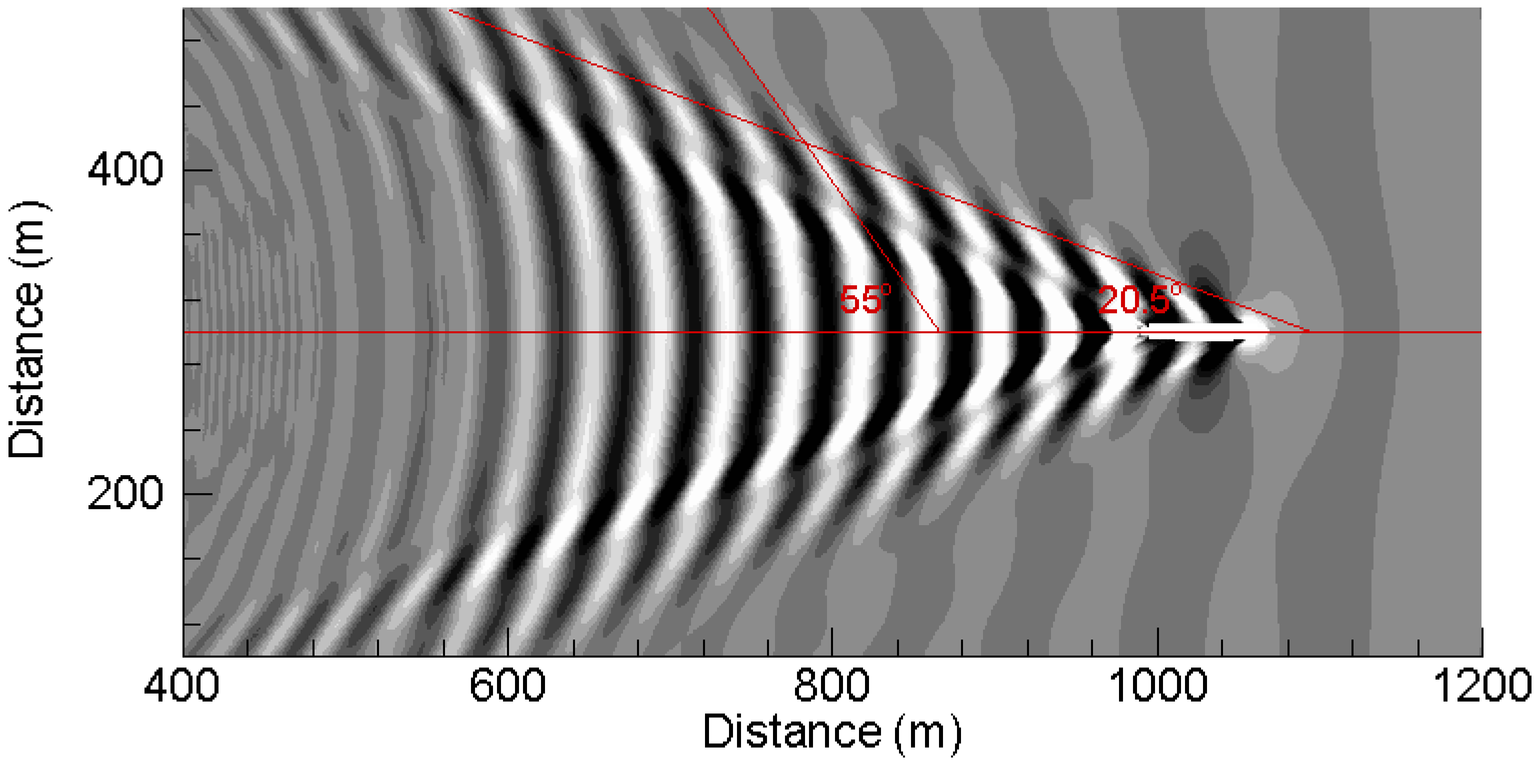
- *Dispersive Limit*
  - *Water Depth/Wavelength, $h/L < 0.5$*
  - *Depth-based Froude Number, $F_{nh} > 0.65$*
- *Wavelength/Grid Size, $L/\Delta x > 10$*
- *Wave Period/Time Step Size, $T/\Delta t > 10$*
- *Courant Number for Numerical Stability*

$$C_r = \sqrt{\left[C^2 \Delta t^2 \left(\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} \right) \right]} \quad < \quad 1$$

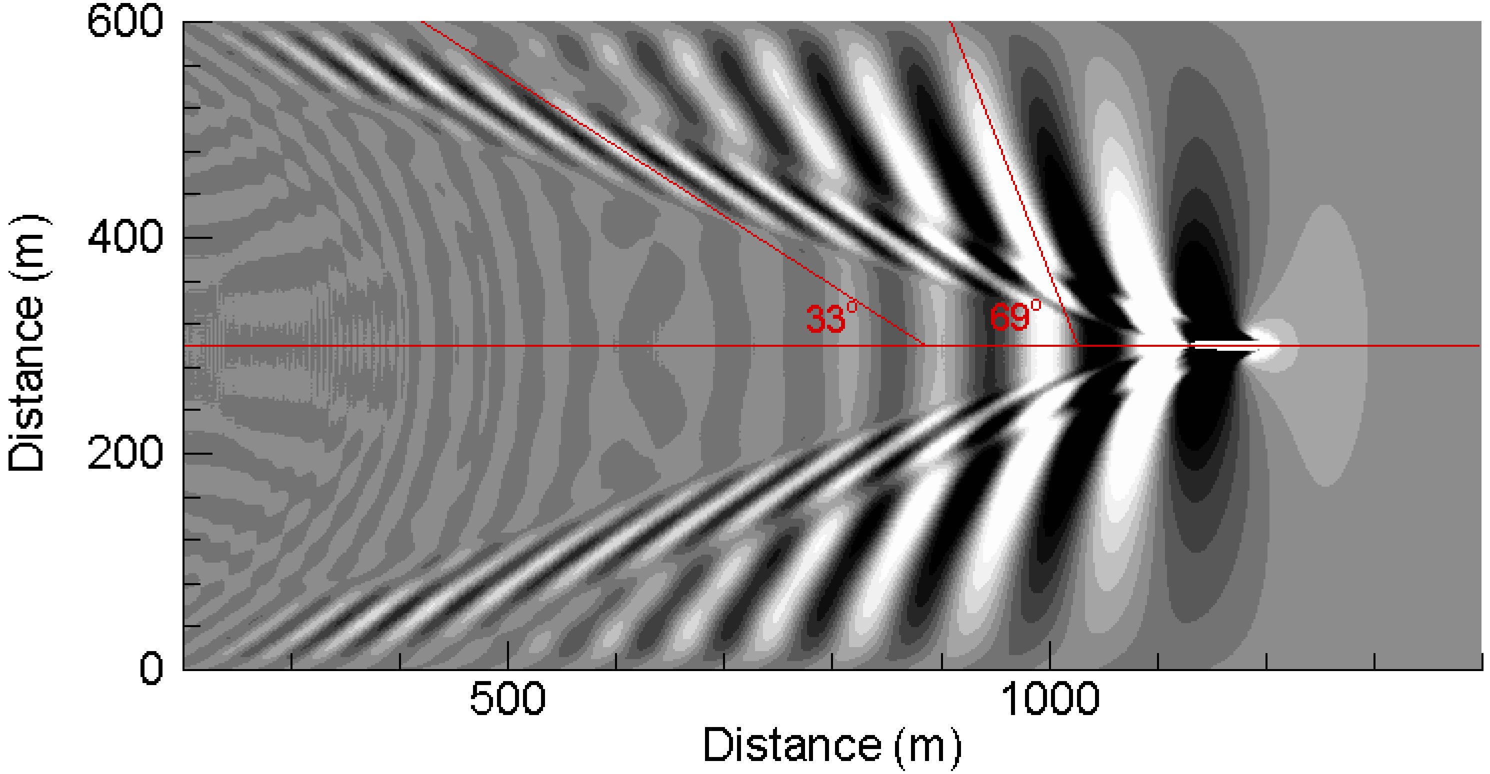
Characteristics of Ship-Generated Waves in Shallow Water

- *Different flow regimes depending on depth based Froude Number* ($F_{nh} = U_s/\sqrt{gh}$)
  - *Subcritical regime* ($F_{nh} < 1$)
  - *Transcritical regime* ($F_{nh} \approx 1$)
  - *Supercritical regime* ($F_{nh} > 1$)

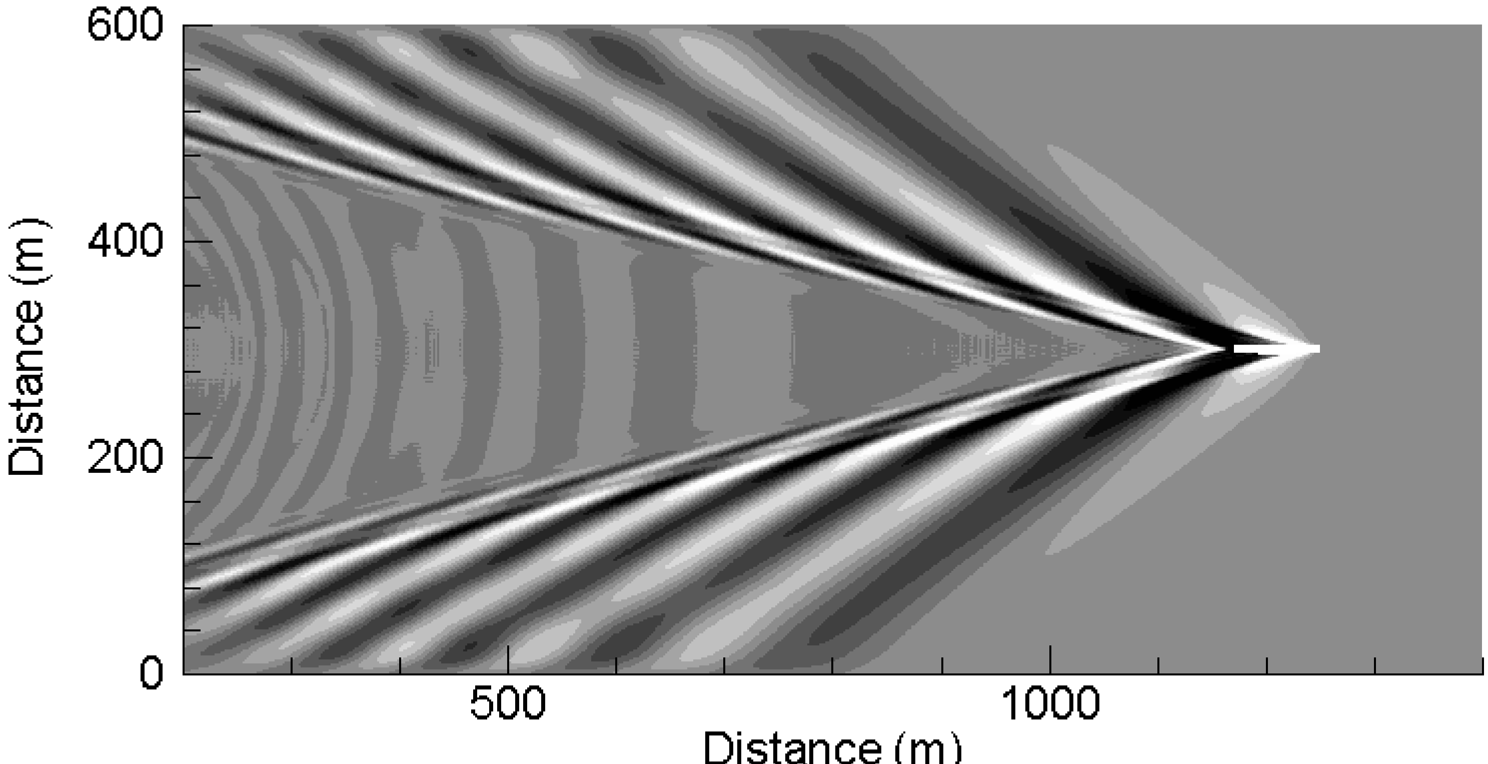
Instantaneous Water Surface Elevation for Ship Moving at Subcritical Speed (F_{nh} = 0.65)

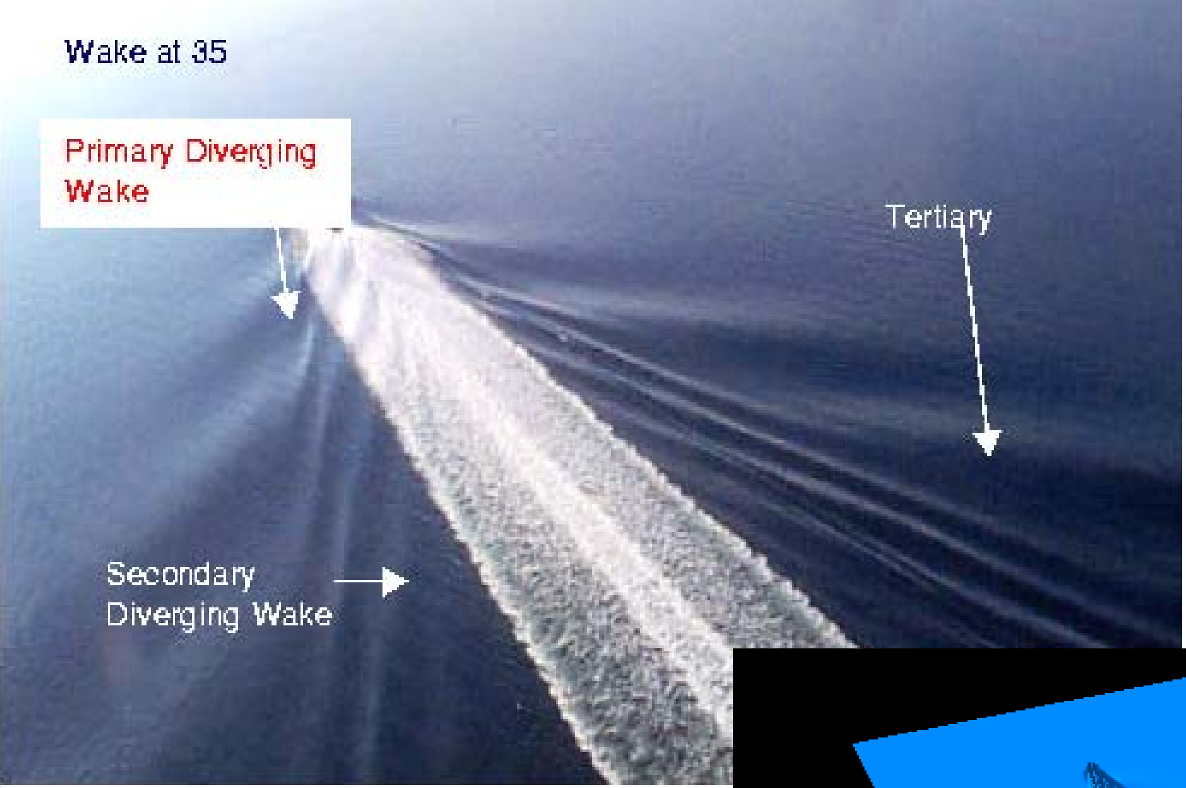


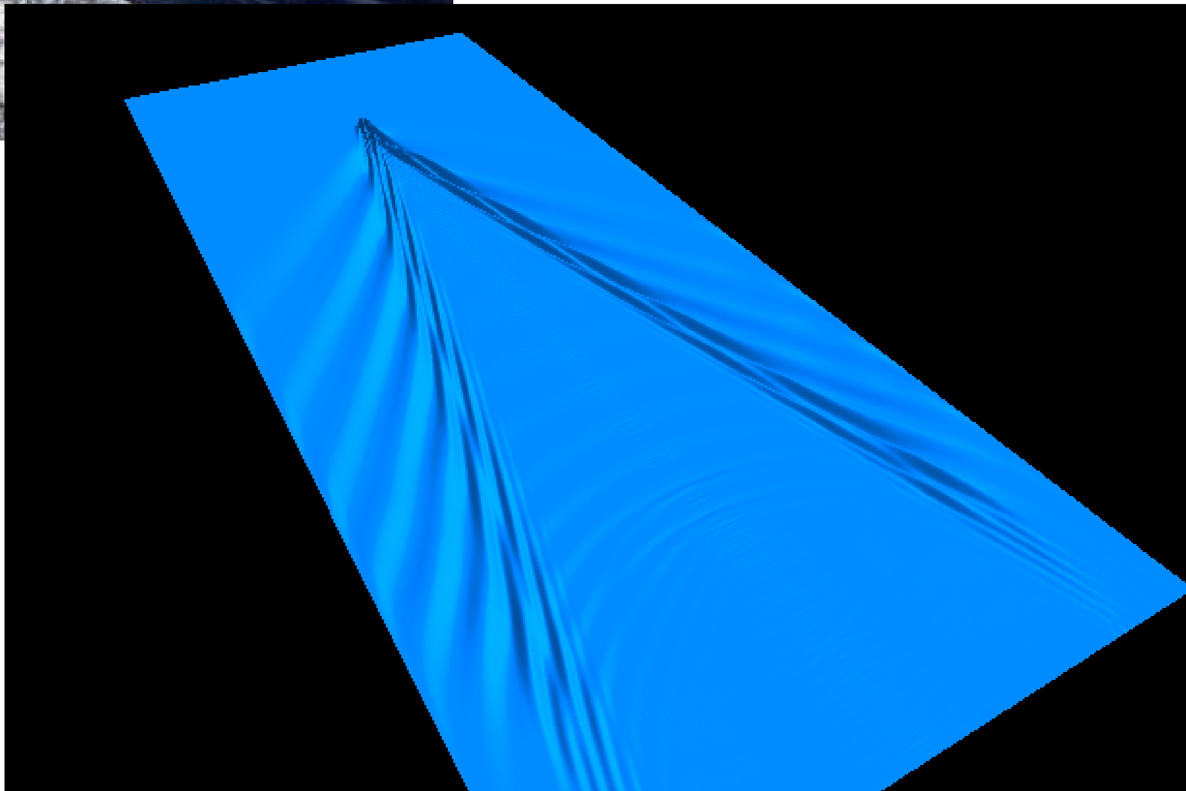
Instantaneous Water Surface Elevation for Ship Moving at Transcritical Speed (F_{nh} = 0.9)



Instantaneous Water Surface Elevation for Ship Moving at Supercritical Speed (F_{nh} = 1.5)




Wake at 35
Primary Diverging Wake
Tertiary
Secondary Diverging Wake



High-Speed Vessel Waves

- *Longer Periods (T = 9.5s for 35knots compared to 4.2s for 15knots)*
  - *Swell-type waves*
  - *Strongly affected by bathymetry/topography of channels*
  - *May not be visible until shoal and break in shallow water*
  - *Different breaking wave type (Plunging)*

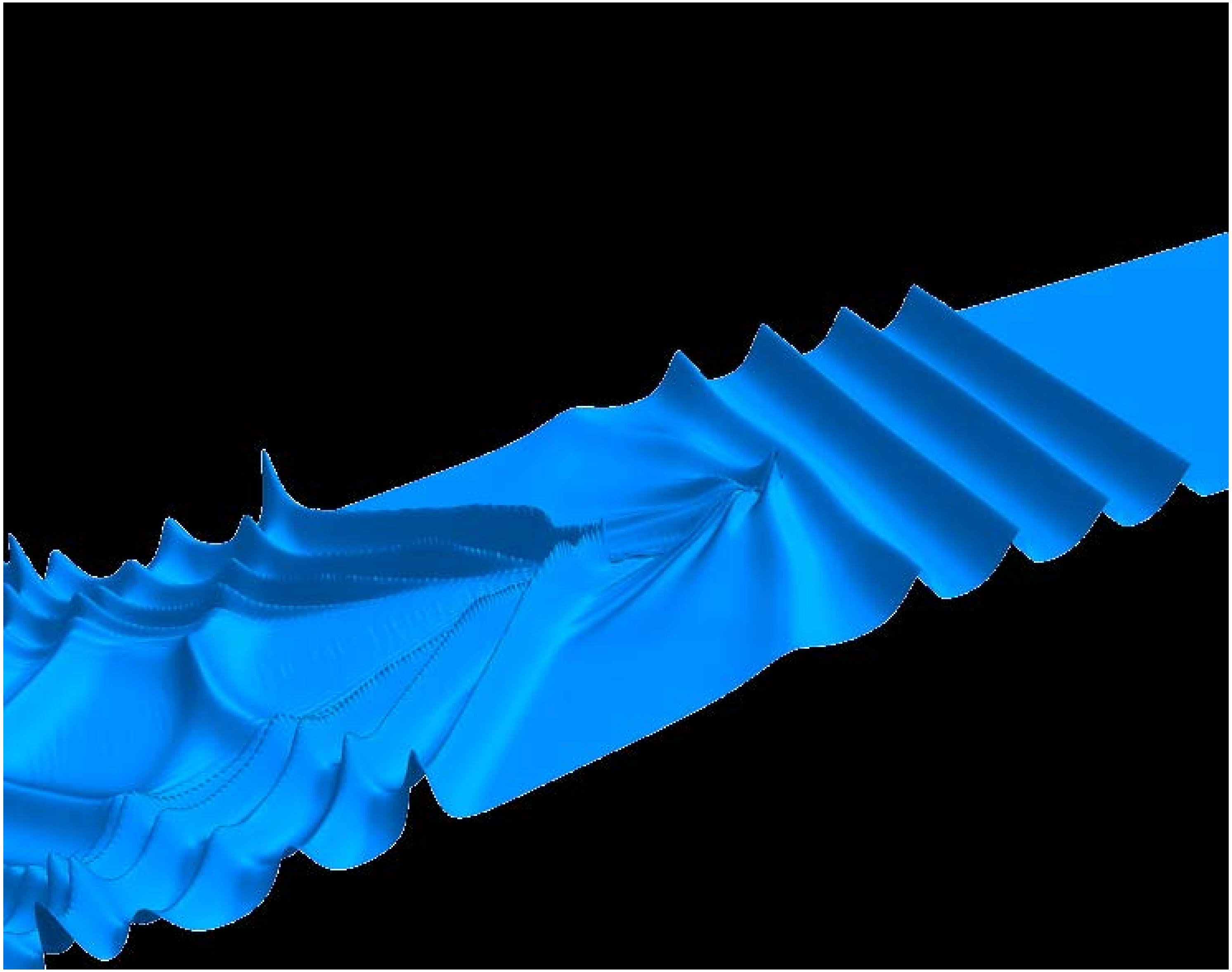
Generation of Upstream-Advancing Solitary Waves

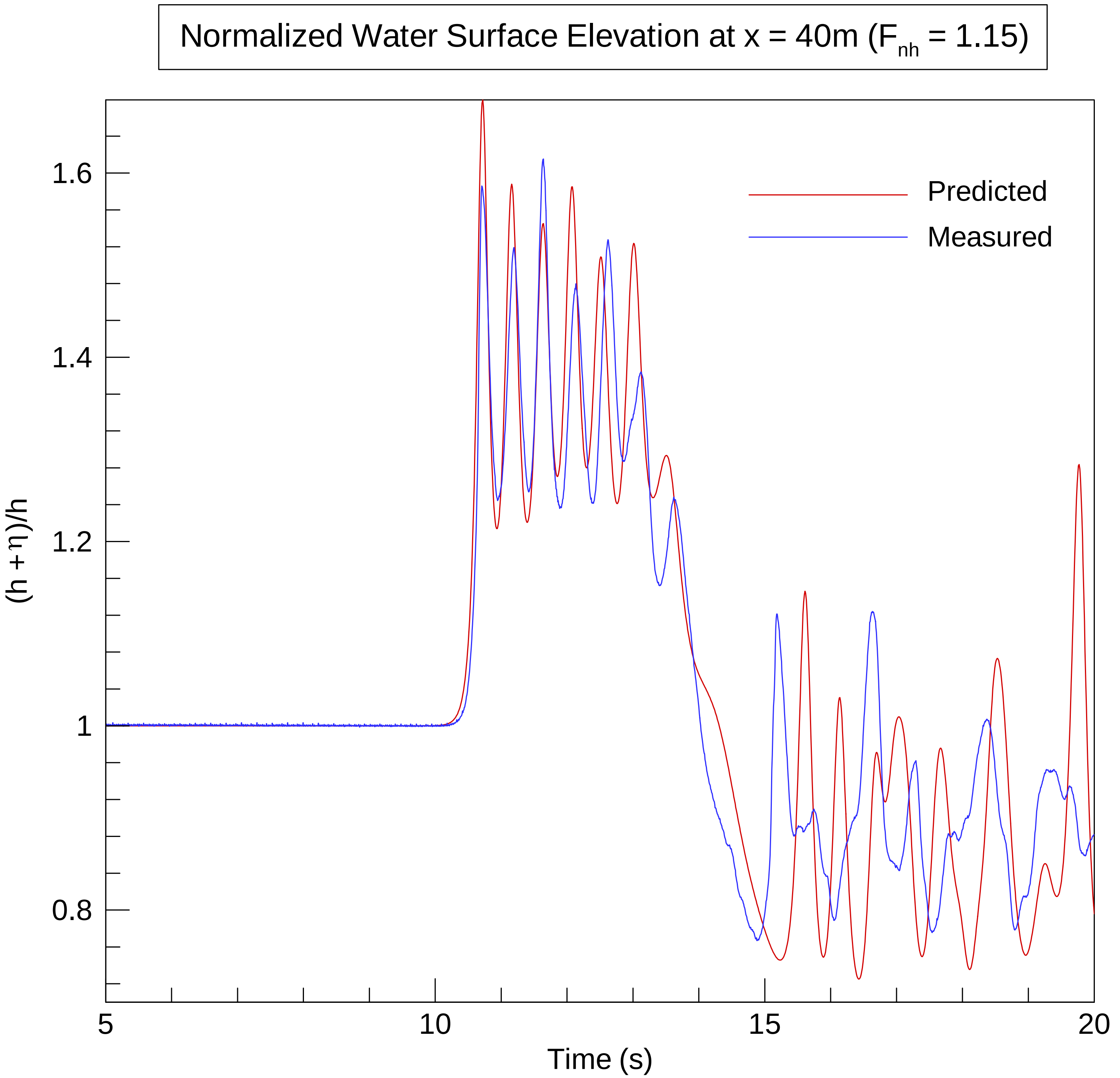
- *Occur at transcritical speeds ($F_{nh} = 1$)*
- *Initial observation by Sir John Scott Russell (1834) in a canal*
- *Maintain shape by balance between nonlinear and dispersive effects*
- *Wave breaking and transition to bores at supercritical speeds*

Laboratory Experiments – Gourlay (2001)

- *60-m long towing tank*
- *Tests with high-speed vessel (L = 1.6m, B = 0.4m , D = 0.1m)*
- *Depth-based Froude numbers from 1.05 to 1.5*

Upstream Solitary Waves (F_{nh} = 1.10)




Normalized Water Surface Elevation at x = 40m (F_{nh} = 1.15)
Predicted
Measured
(h + η)/h
Time (s)
1.6
1.4
1.2
1
0.8
5
10
15
20

Deep-Draft Vessels in Navigation Channels

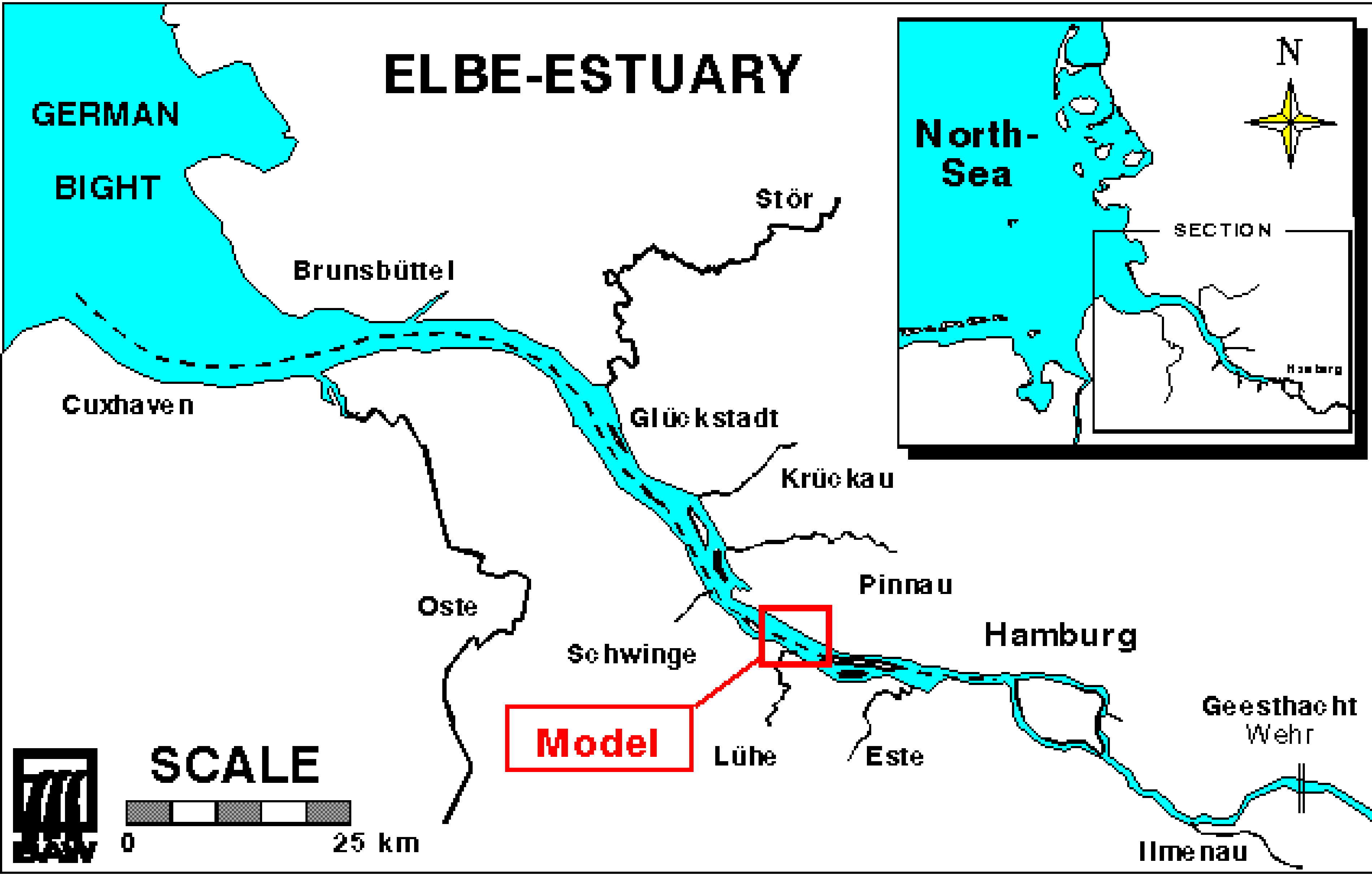
- *Water Level Drawdown / Return Velocity in Confined Channels*
- *Underkeel Clearance (Squat / Vessel Motions)*
- *Generation of Surge Waves on Shallow Banks*

Water Level Drawdown and Return Currents in Confined Channels

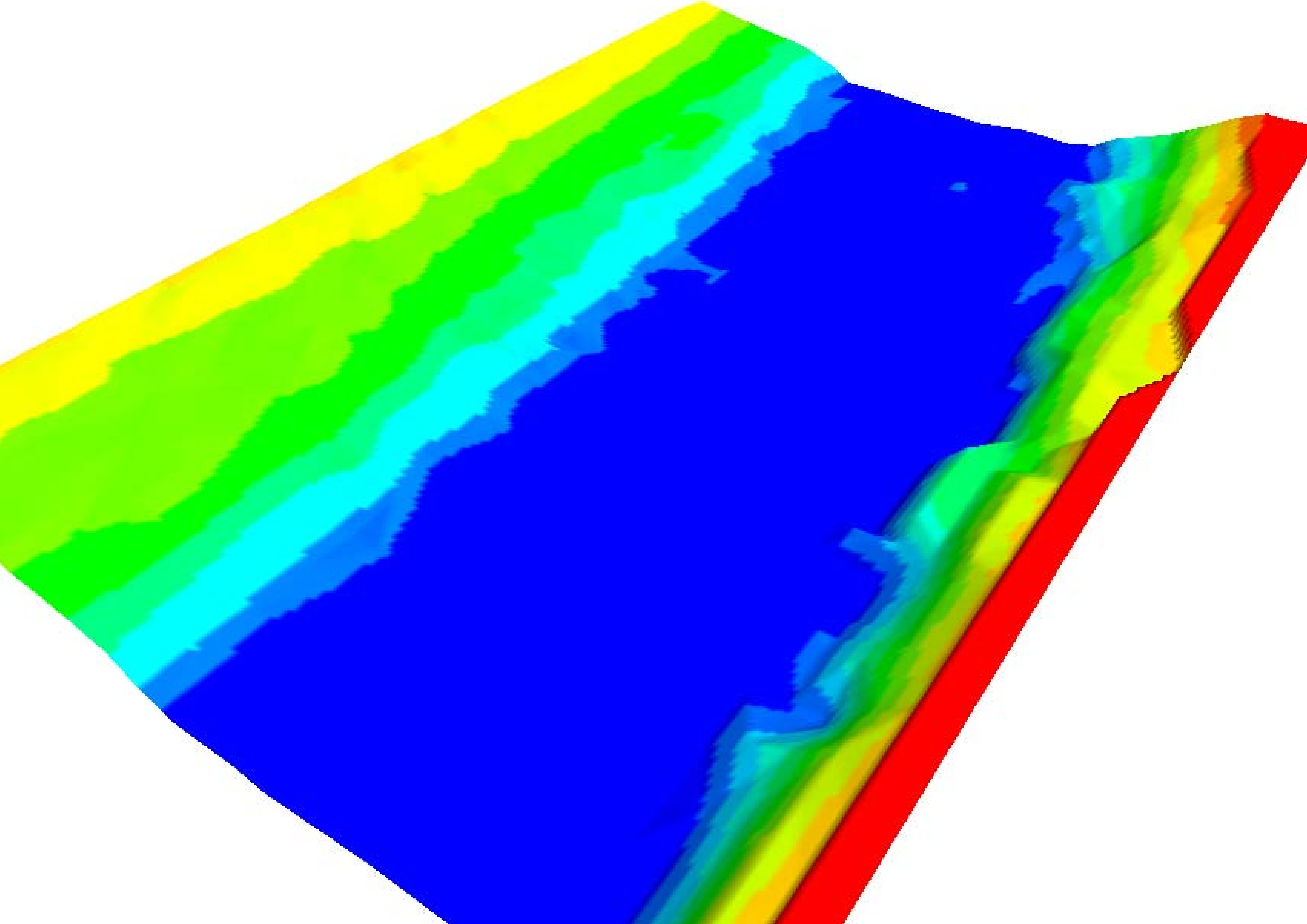
- *1-D Analytical Model - Schijf (1949)*
  - *based on mass and energy conservation equations, does not consider across-channel variations in bottom topography or return currents*
- *Does not apply to 2-D waterways with side banks, structures, islands, etc*

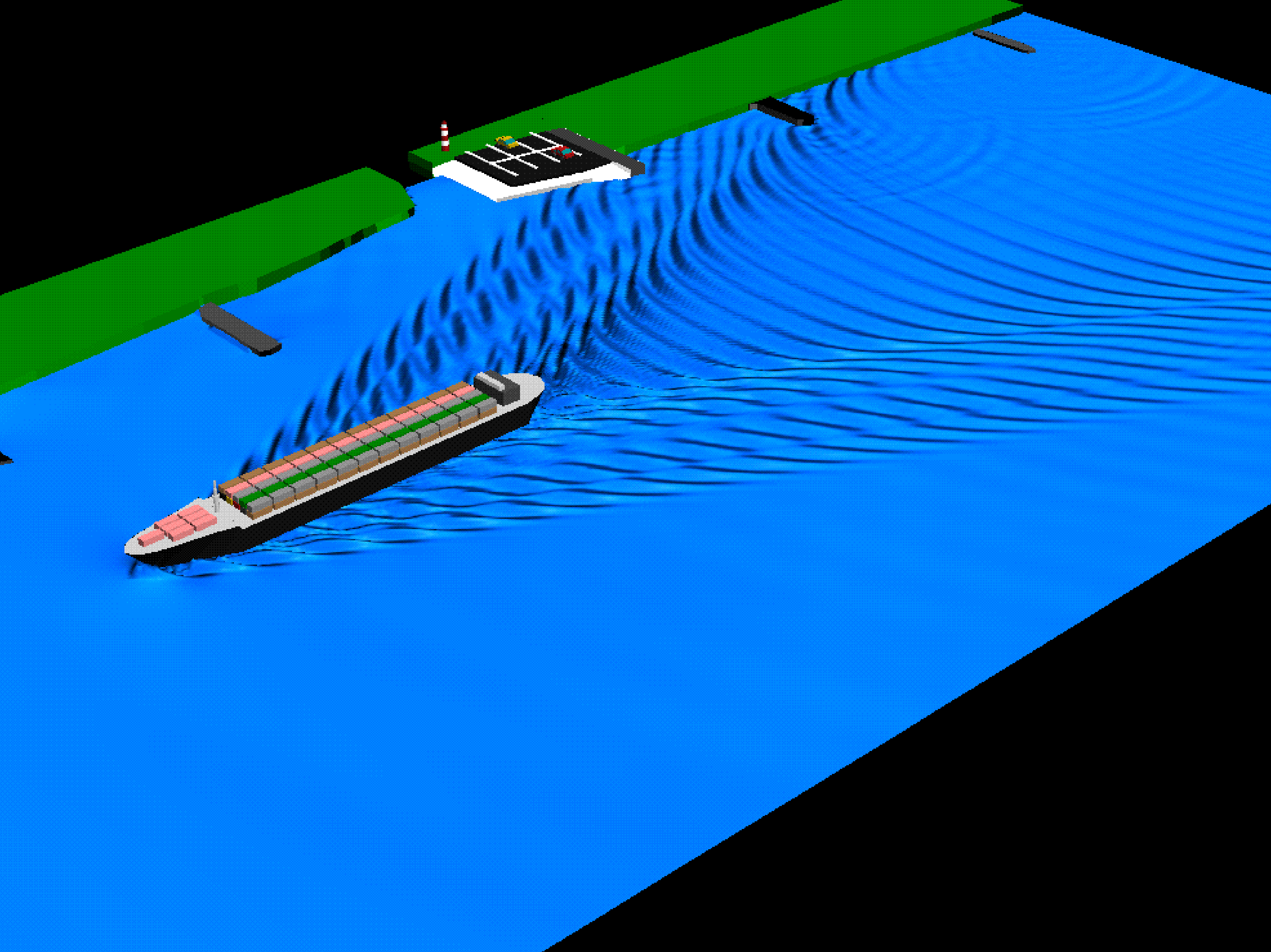
Elbe River Study

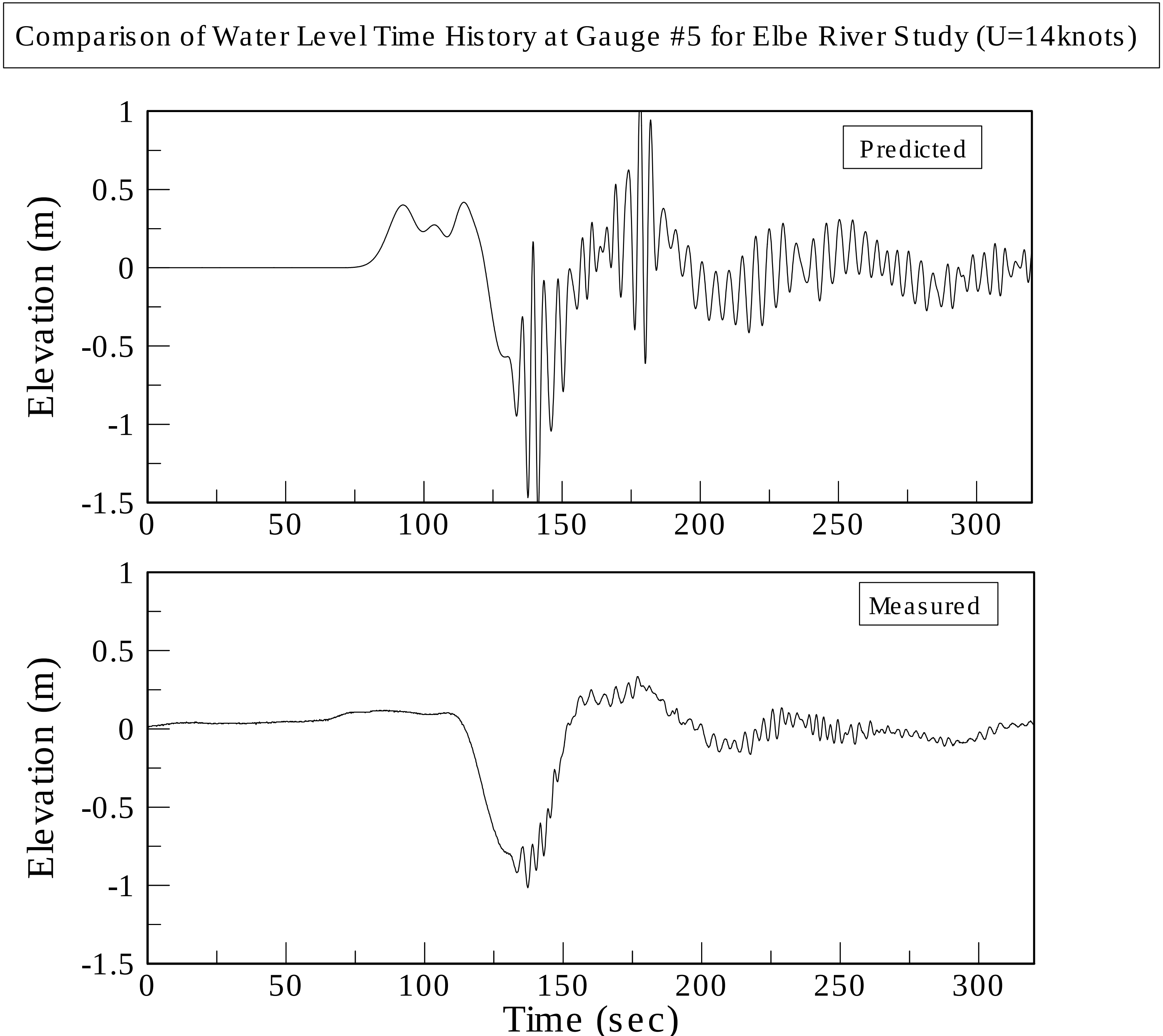
- *60000DWT Panamax Class Vessel*
  - *Length = 280m*
  - *Beam = 32m*
  - *Draft = 13.5m*
- *Ship Speeds (7 to 15 knots)*
- *1:40 Physical Model Study at German Waterways Department*


ELBE-ESTUARY
GERMAN
BIGHT
North-
Sea
N
SECTION
Brunsbüttel
Stör
Cuxhaven
Glückstadt
Krückau
Oste
Pinnau
Hamburg
Schwinge
Model
Lühe
Este
Geesthacht
Wehr
SCALE
0
25 km
Ilmenau

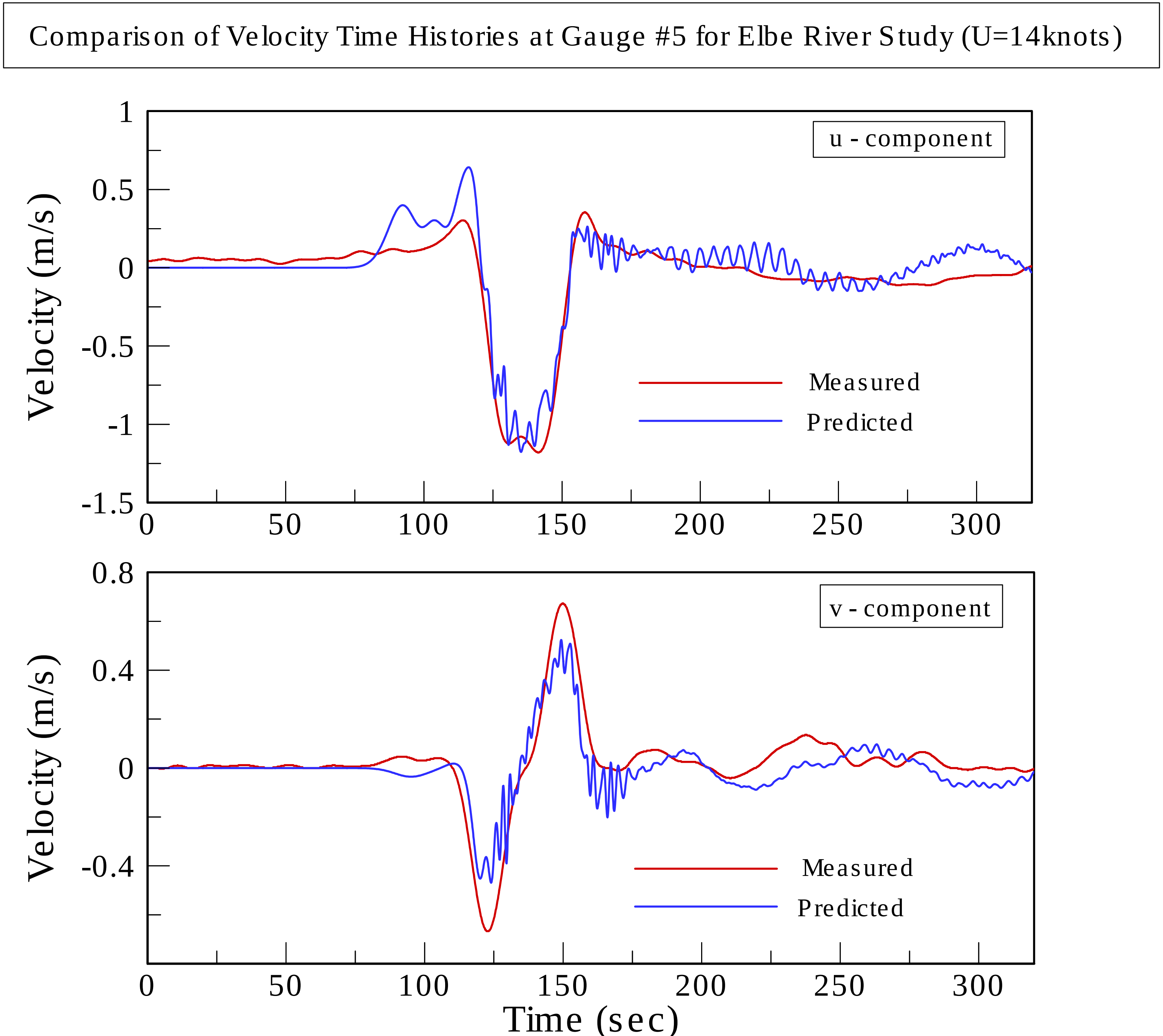
ELBE-B1e.CH9







Comparison of Water Level Time History at Gauge #5 for Elbe River Study (U=14knots)
Predicted
Measured
Elevation (m)
Time (sec)
1
0.5
0
-0.5
-1
-1.5
0
50
100
150
200
250
300



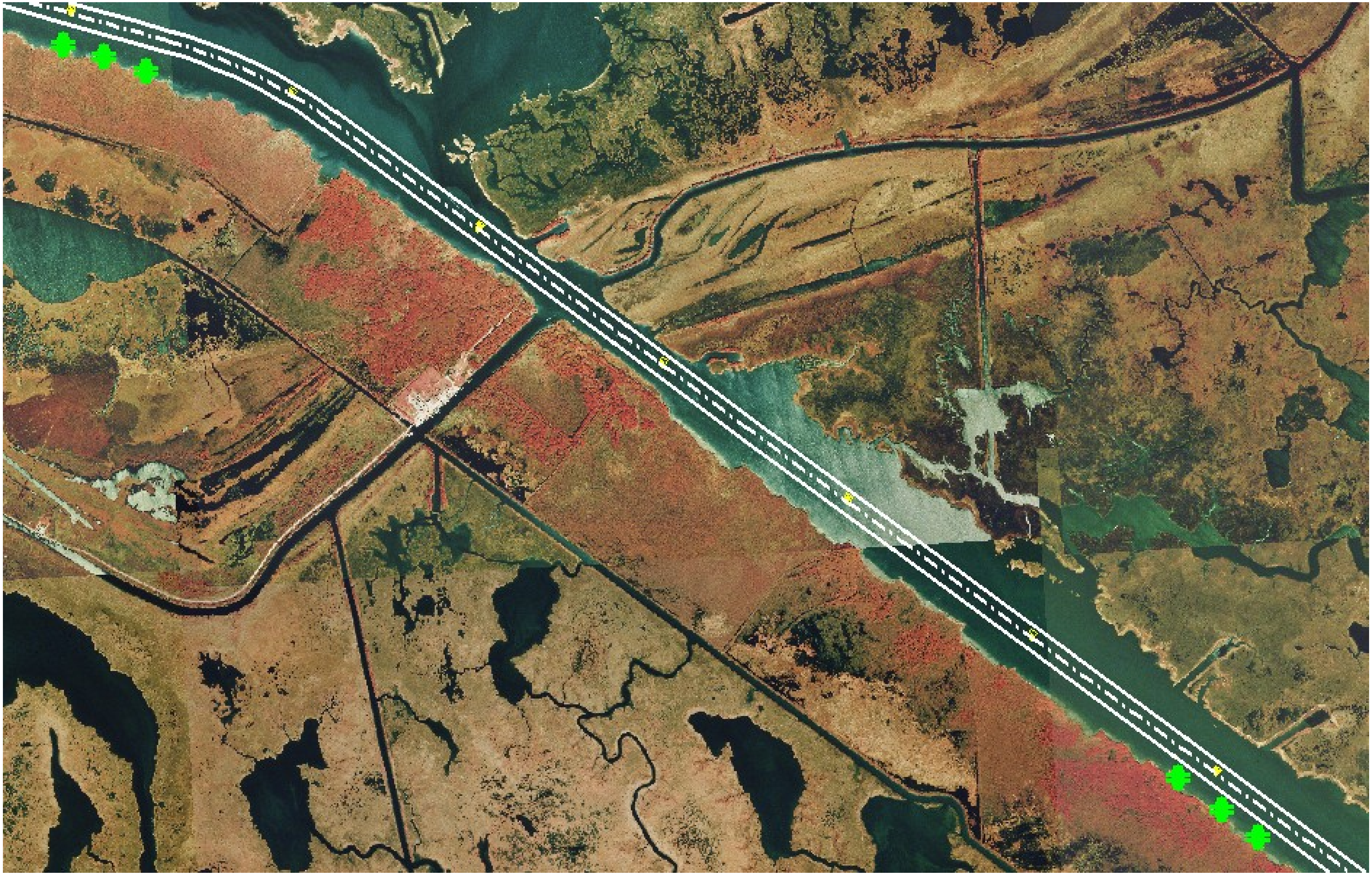
Comparison of Velocity Time Histories at Gauge #5 for Elbe River Study (U=14knots)
u - component
Measured
Predicted
Velocity (m/s)
1
0.5
0
-0.5
-1
-1.5
0
50
100
150
200
250
300
v - component
Measured
Predicted
Velocity (m/s)
0.8
0.4
0
-0.4
0
50
100
150
200
250
300
Time (sec)

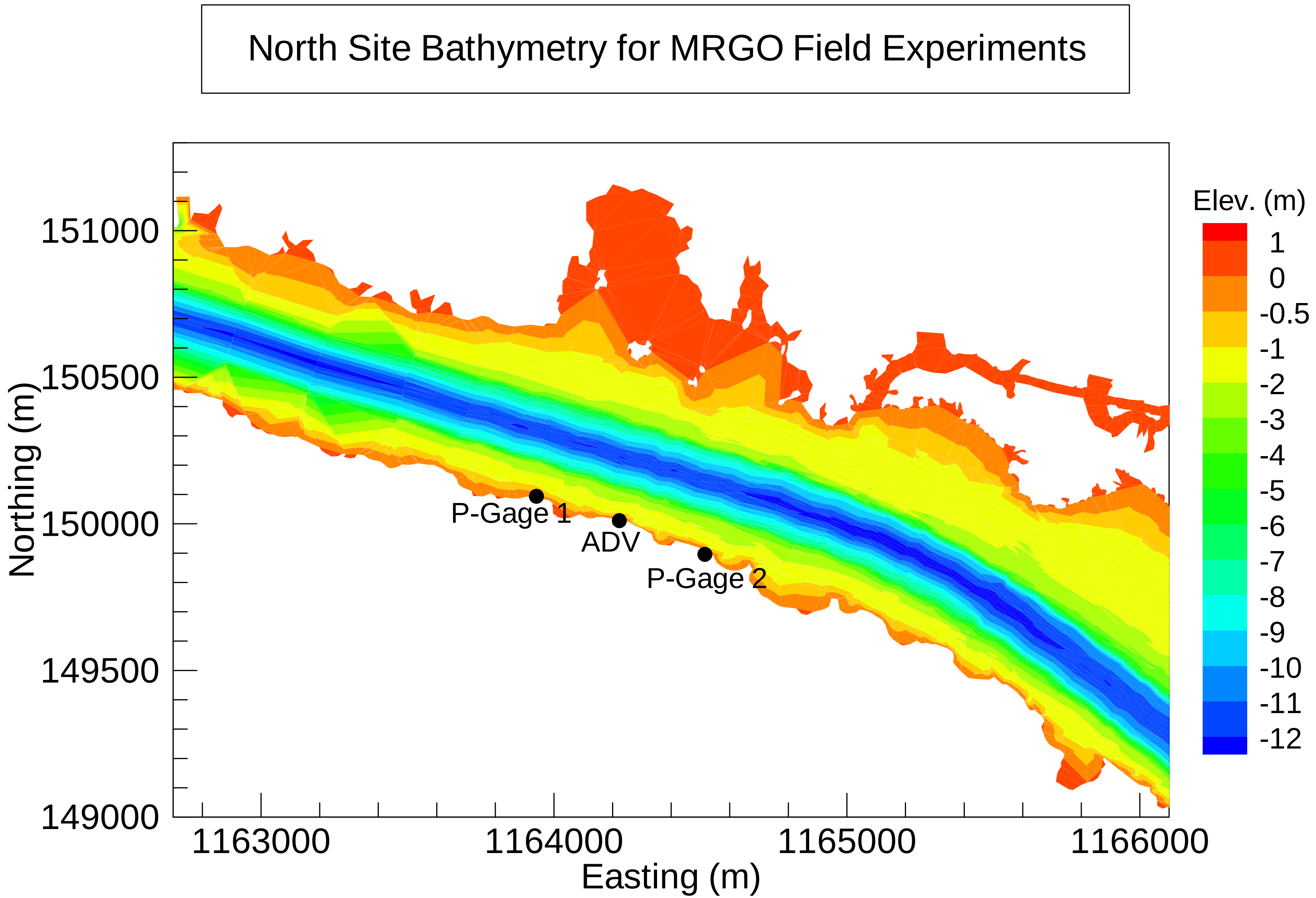
Mississippi River – Gulf Outlet (MRGO) Channel

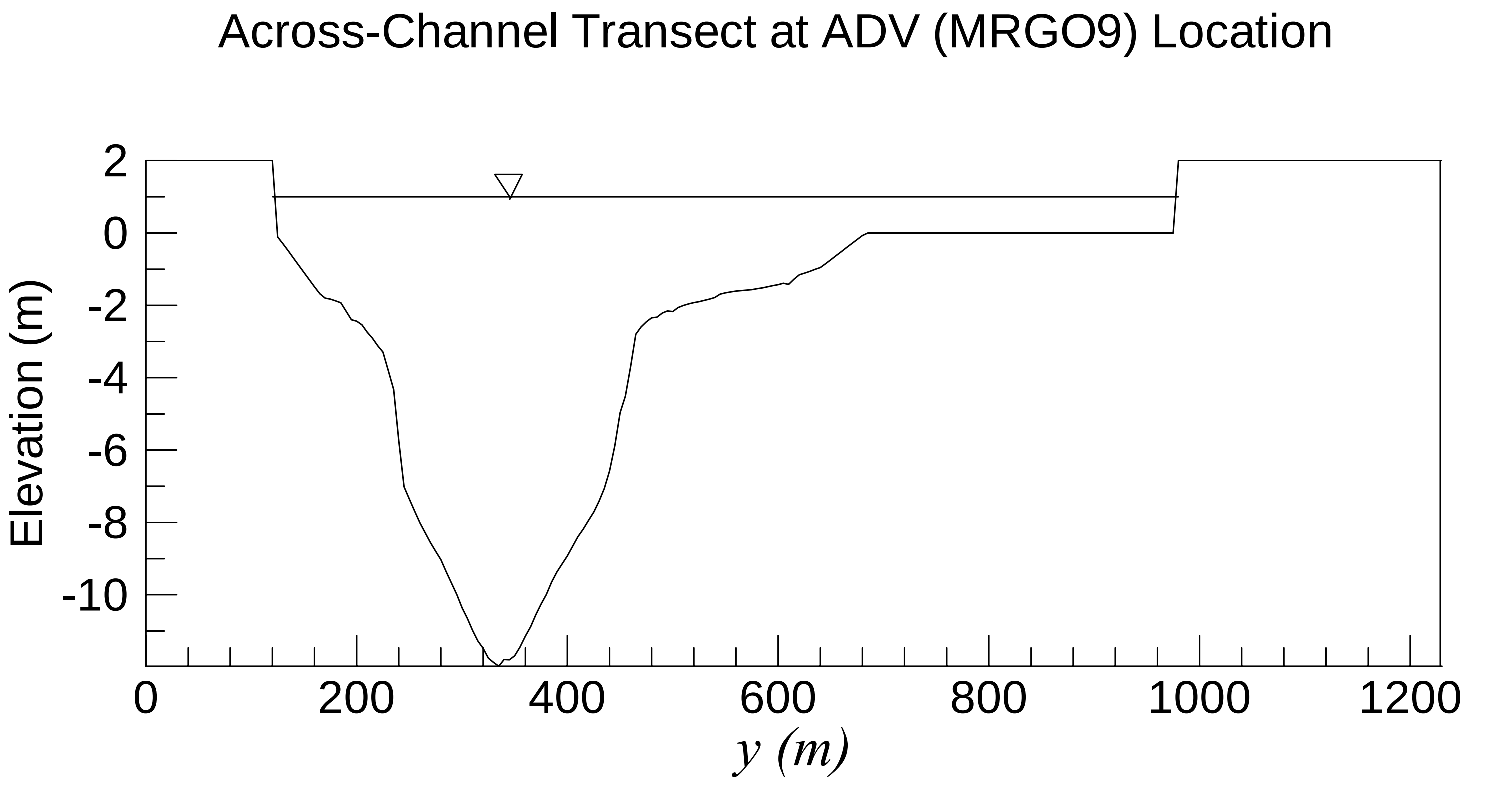
- *76 miles long, 500 ft wide and 40 ft deep*
- *Northern banks eroding at an average rate of 15 feet per year*
- *Considering use of articulated concrete mats to protect the channel banks*

MRGO Field Experiments

- *Measurements: 30 May to 10 June 2002*
- *Four pressure gages*
- *Two Acoustic Doppler Velocimeters (ADVs)*
- *Deployed at two locations along channel banks (33000 ft apart)*




North Site Bathymetry for MRGO Field Experiments
Elev. (m)
1
0
-0.5
-1
-2
-3
-4
-5
-6
-7
-8
-9
-10
-11
-12
P-Gage 1
ADV
P-Gage 2
151000
150500
150000
149500
149000
Northing (m)
1163000
1164000
1165000
1166000
Easting (m)



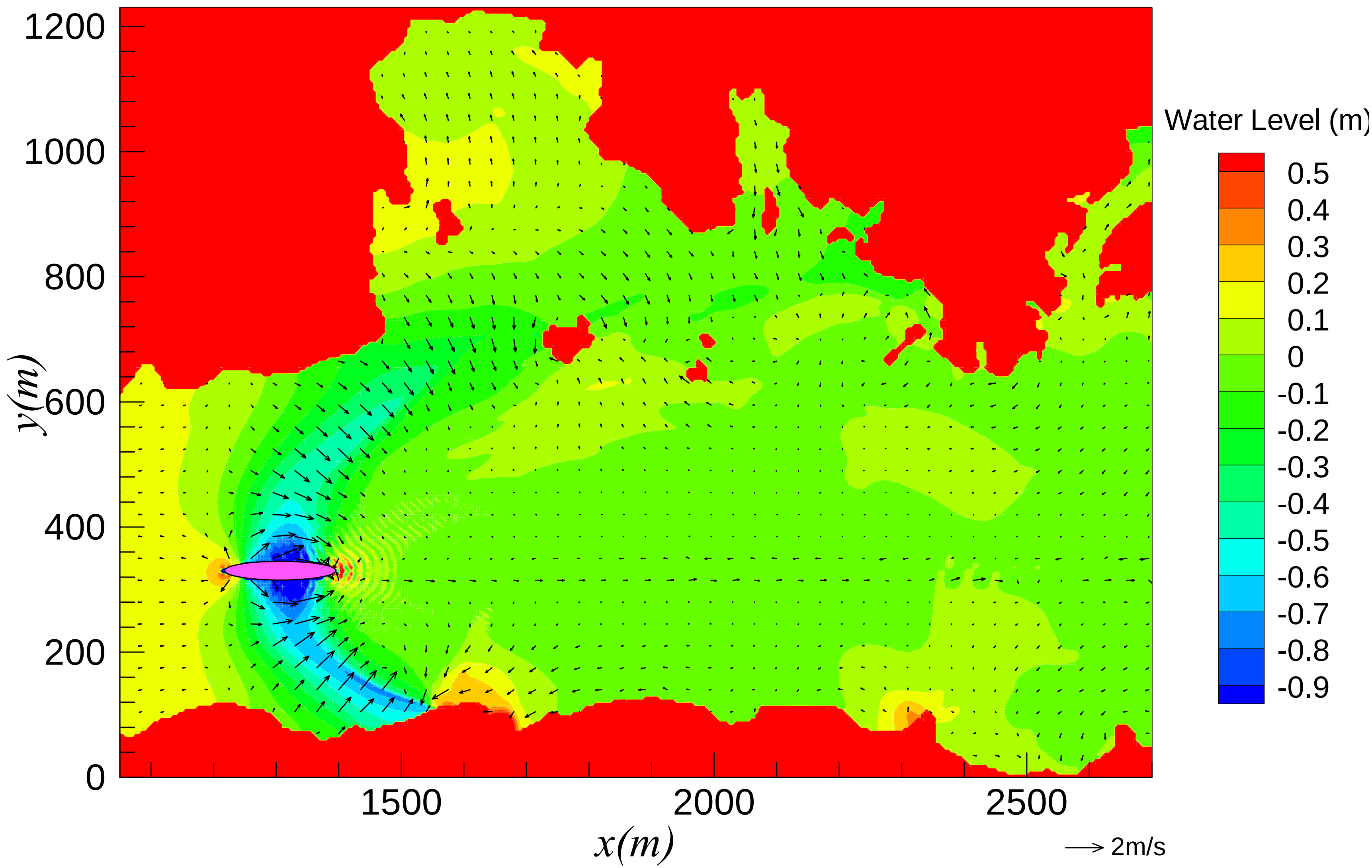
Across-Channel Transect at ADV (MRGO9) Location
Elevation (m)
2
0
-2
-4
-6
-8
-10
0
200
400
600
800
1000
1200
y (m)

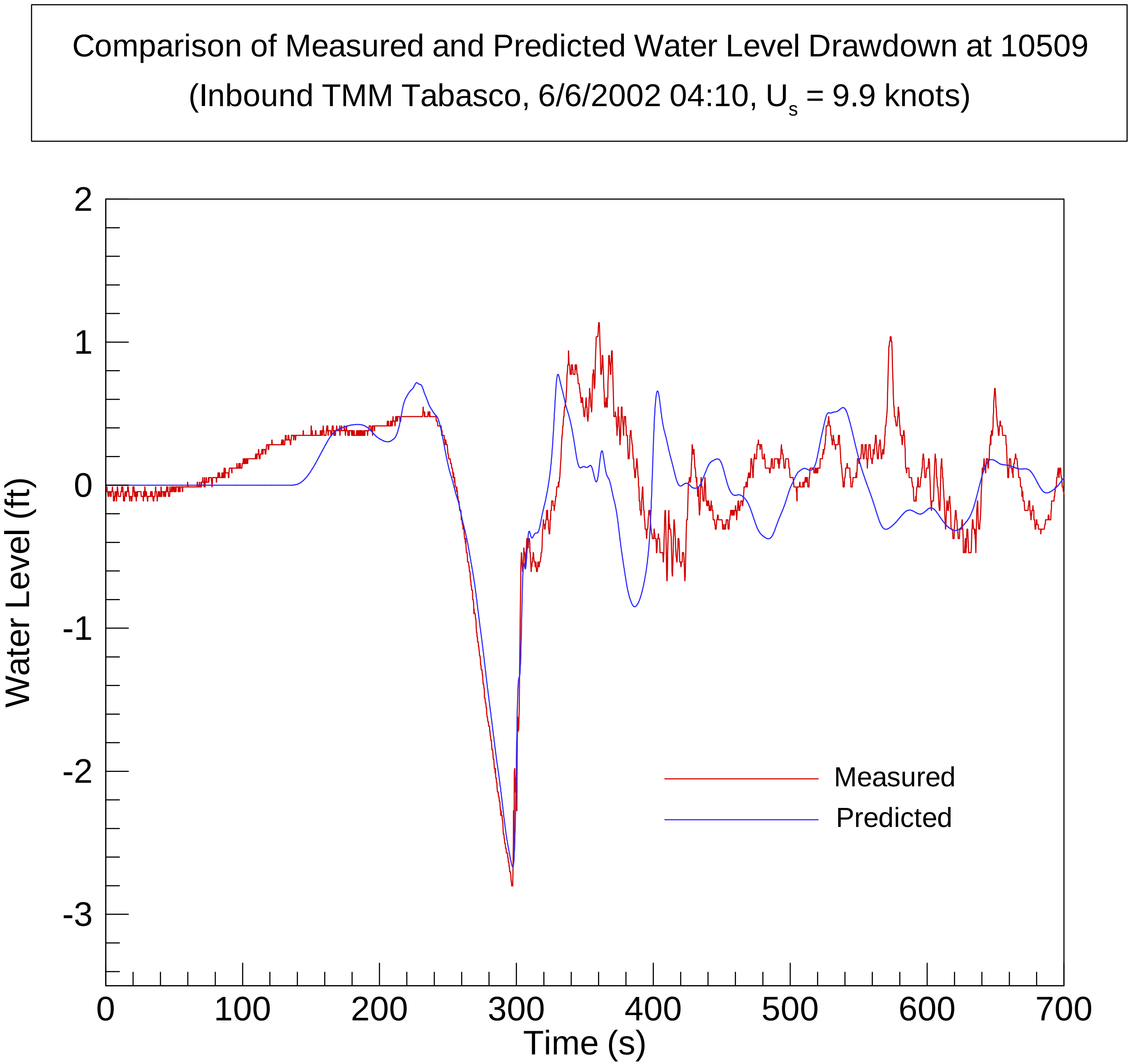
Sample Measured Data

- *5/30/02 at 21:00 hrs*
  - *Libra Buenos Aires (Length = 195m, Beam = 30m, Draft = 10m)*
  - *Estimated Ship Speed = 12knots*
- *6/9/02 at 23:45 hrs*
  - *Lykes Commander (Length = 188m, Beam = 32m, Draft = 12.5m)*
  - *Estimated Ship Speed = 10knots*

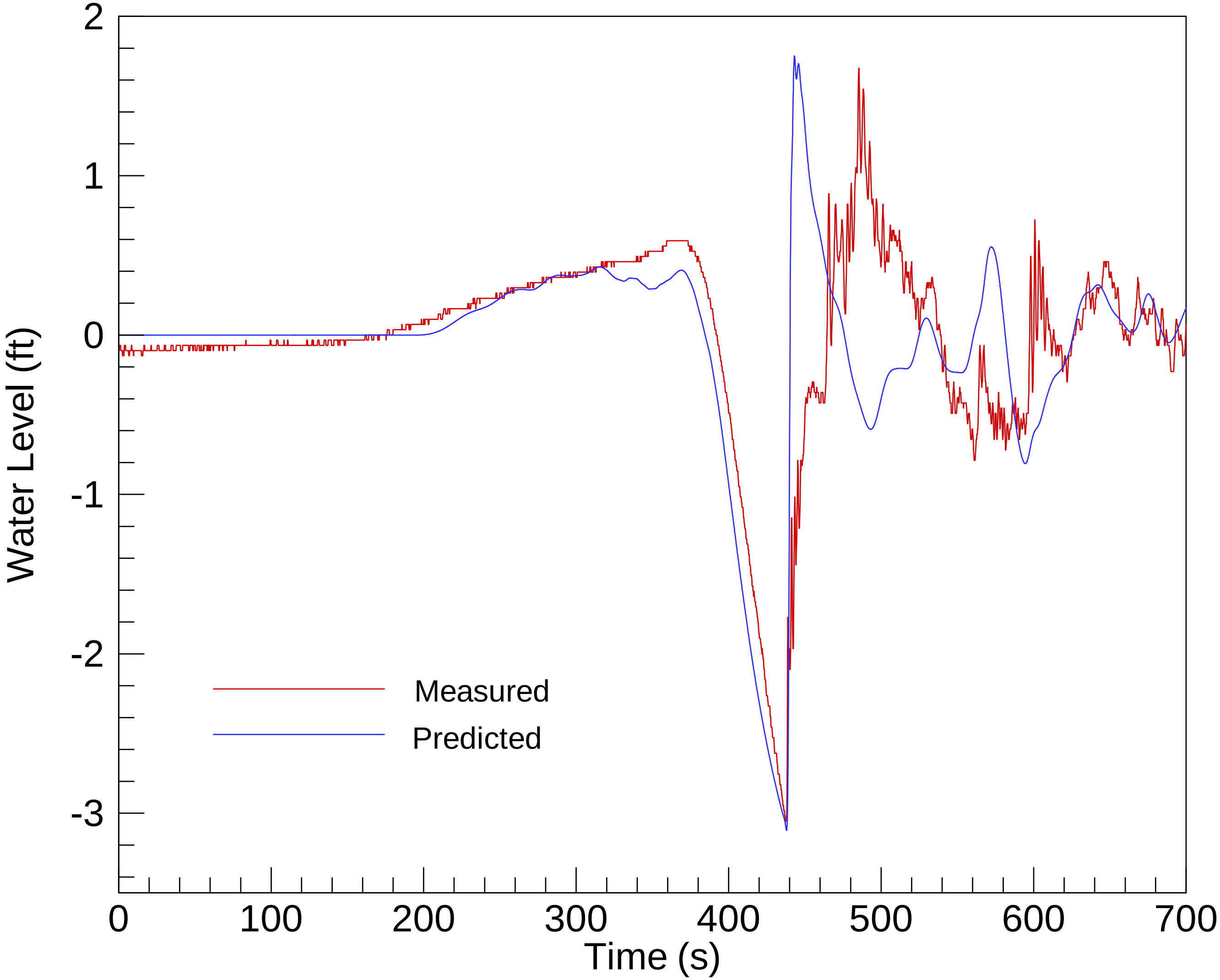
Ship-Induced Water Level and Flow Field Pattern

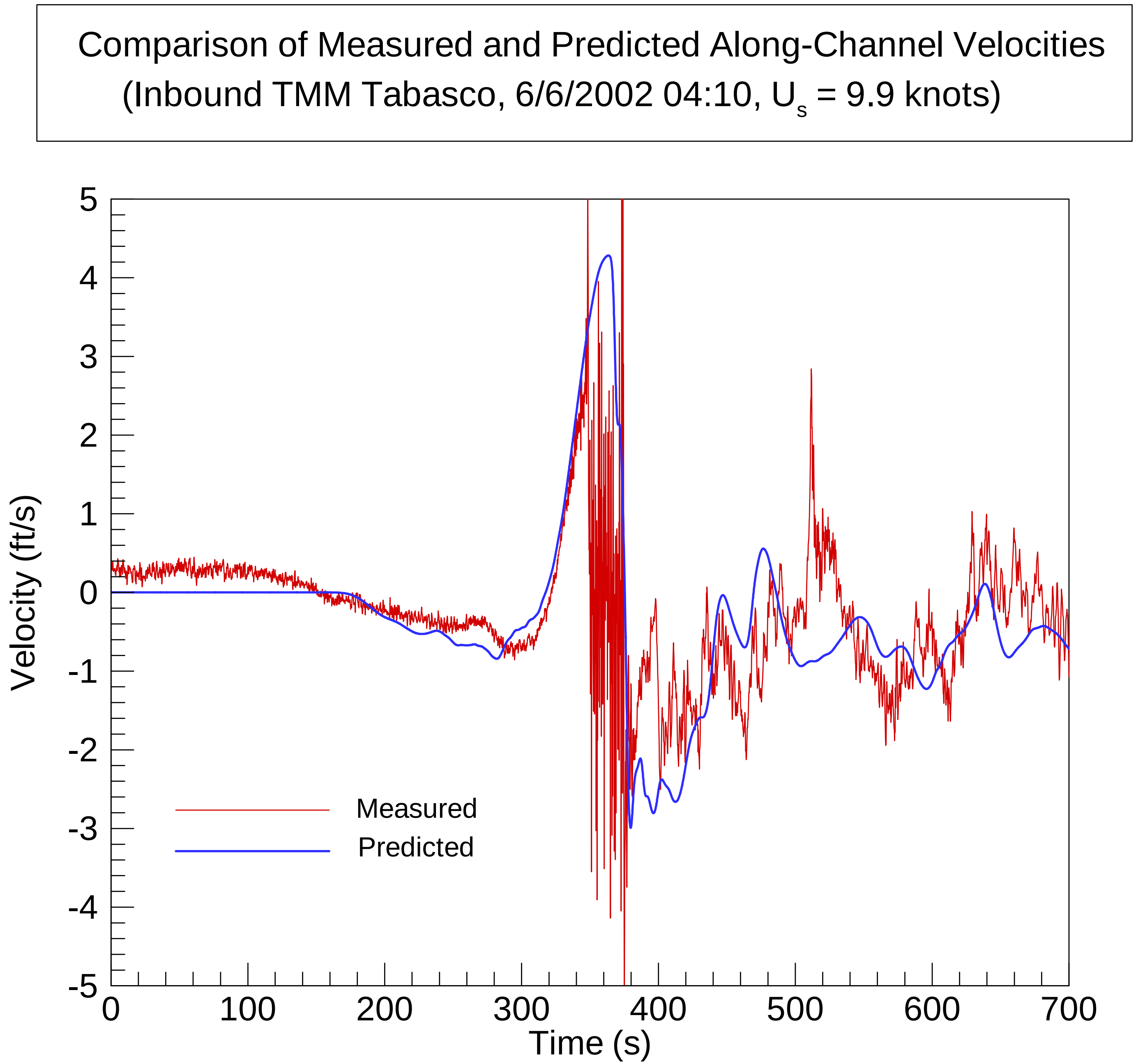
(Inbound TMM Tabasco, 6/6/2002 04:10, U_s = 9.9 knots)

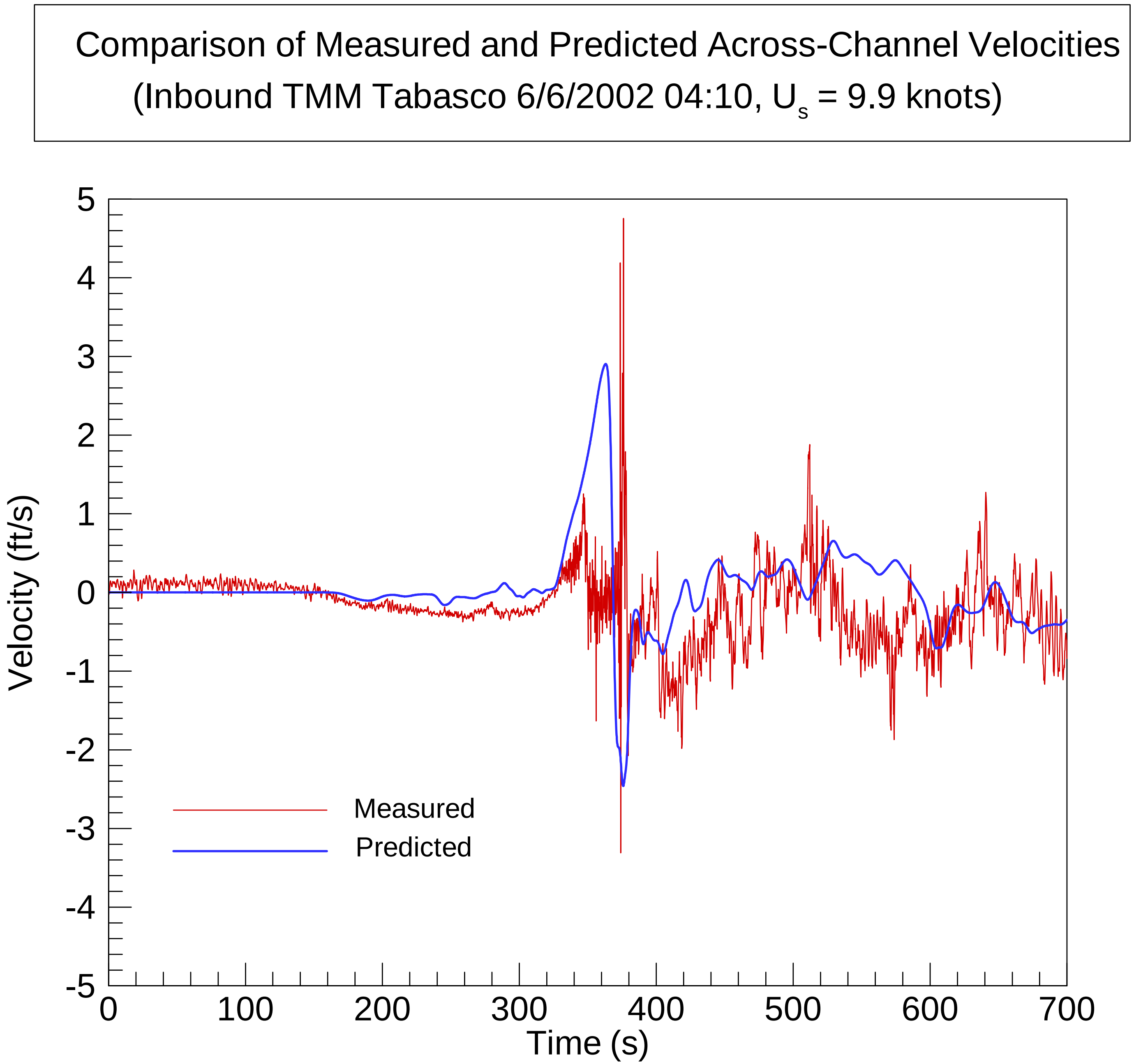



Comparison of Measured and Predicted Water Level Drawdown at 10509
(Inbound TMM Tabasco, 6/6/2002 04:10, U$_s$ = 9.9 knots)
Water Level (ft)
Time (s)
2
1
0
-1
-2
-3
0
100
200
300
400
500
600
700
Measured
Predicted

Comparison of Measured and Predicted Water Level Drawdown at 10508
(Inbound TMM Tabasco, 6/6/2002 04:10, U_s = 9.9 knots)




Comparison of Measured and Predicted Along-Channel Velocities
(Inbound TMM Tabasco, 6/6/2002 04:10, U_s = 9.9 knots)
Velocity (ft/s)
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
100
200
300
400
500
600
700
Time (s)
Measured
Predicted



Comparison of Measured and Predicted Across-Channel Velocities
(Inbound TMM Tabasco 6/6/2002 04:10, U_s = 9.9 knots)
Velocity (ft/s)
5
4
3
2
1
0
-1
-2
-3
-4
-5
0
100
200
300
400
500
600
700
Time (s)
Measured
Predicted

Concluding Remarks

- *Boussinesq models are promising for investigating ship-generated waves in shallow water and their interaction with river banks and moored vessels*
- *Couple Boussinesq model with bank erosion and 6-dof vessel motion models*
- *Solve Boussinesq equations on unstructured grid using finite volume method*